

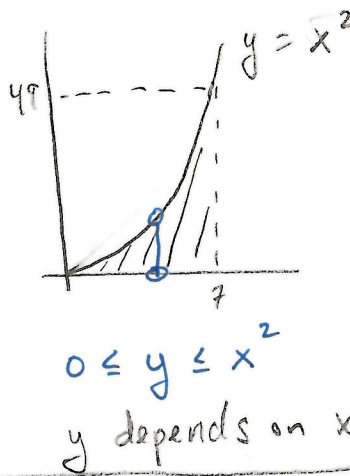
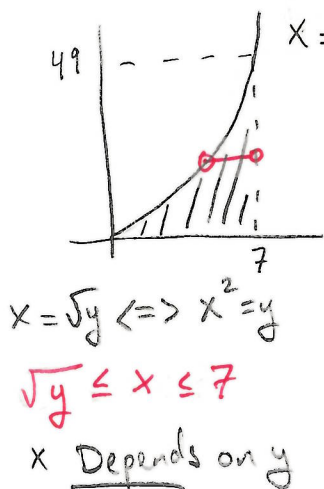
QUIZ 22 SOLUTIONS: LESSON 28
APRIL 3, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

Let $f(x, y) = \sqrt{x^3 + 1}$ and consider the region R described by

$$\begin{cases} \sqrt{y} \leq x \leq 7 \\ 0 \leq y \leq 49 \end{cases}$$

1. [3 pts] Describe the region R so that x is the **independent** variable and y is the **dependent** variable.



Same region, different description

$$R = \begin{cases} 0 \leq y \leq x^2 \\ 0 \leq x \leq 7 \end{cases}$$

2. [2 pts] Swap the order of integration of

$$\int_0^{49} \int_{\sqrt{y}}^7 \sqrt{x^3 + 1} \, dx \, dy = \int_0^7 \int_0^{x^2} \sqrt{x^3 + 1} \, dy \, dx$$

x dependent
y independent

y dependent
x independent

MATH 16020 QUIZ 22 SOLUTIONS

3. [5 pts] Compute the integral you found in # 2. Round your answer to 2 decimal places.

$$\int_0^7 \int_0^{x^2} \sqrt{x^3+1} \, dy \, dx = \int_0^7 y \sqrt{x^3+1} \Big|_{y=0}^{y=x^2} dx$$

$$\begin{aligned} u &= x^3+1 \\ du &= 3x^2 dx \\ \Rightarrow dx &= \frac{du}{3x^2} \end{aligned}$$

$$= \int_0^7 x^2 \sqrt{x^3+1} \, dx$$

$$= \int_0^7 x^2 (x^3+1)^{1/2} dx$$

$$= \int_{u(0)}^{u(7)} x^2 (u)^{1/2} \left(\frac{du}{3x^2} \right)$$

$$= \int_{u(0)}^{u(7)} \frac{1}{3} u^{1/2} du$$

$$= \frac{1}{3} \left(\frac{1}{1+1/2} \right) u^{1/2+1} \Big|_{u(0)}^{u(7)}$$

$$= \frac{1}{3} \left(\frac{2}{3} \right) u^{3/2} \Big|_{u(0)}^{u(7)}$$

$$= \frac{2}{9} u^{3/2} \Big|_{u(0)}^{u(7)}$$

$$= \frac{2}{9} (x^3+1)^{3/2} \Big|_0^7$$

$$= \frac{2}{9} (7^3+1)^{3/2} - \frac{2}{9} (0^3+1)^{3/2}$$

$$= \frac{2}{9} (344)^{3/2} - \frac{2}{9}$$

$$\approx \boxed{1417.61}$$

$$\begin{aligned} 1 + \frac{1}{2} &= \frac{2}{2} + \frac{1}{2} = \frac{3}{2} \\ \frac{1}{1 + \frac{1}{2}} &= \frac{1}{\frac{3}{2}} = \frac{2}{3} \end{aligned}$$